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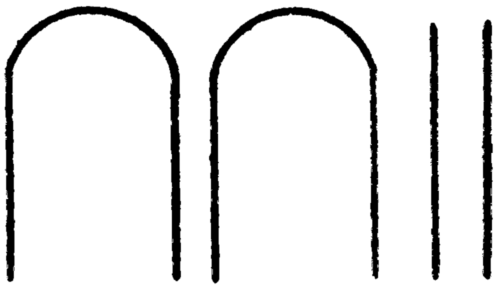
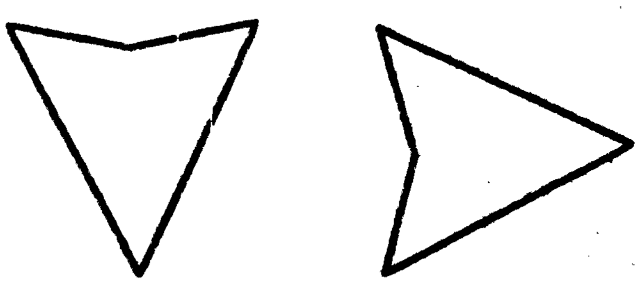
This booklet, one of a series, has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include (1) systems of numeration from an historical point of view, (2) a problem of application in a different number base, and (3) addition and multiplication in base two and five. (RP)



$(324)_{\text{FIVE}}$

NUMERATION SYSTEMS

— PAST AND PRESENT —



$(1001)_{\text{TWO}}$

ESEA Title III

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NUMERATION SYSTEMS

Past and Present

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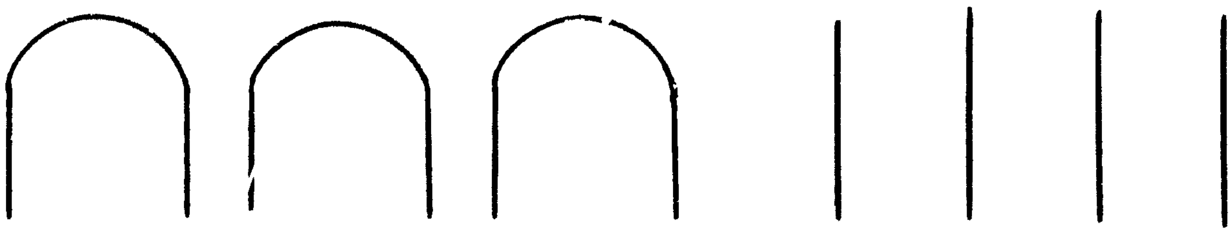
SYSTEMS OF NUMERATION

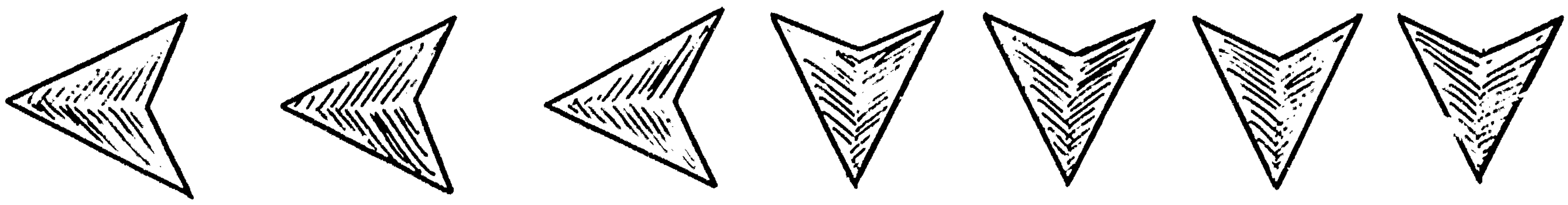
--An Historical View

A scout spotted an enemy patrol which was approaching his camp. He must report this to his leader immediately. He "counted" the number of enemy soldiers by making <u>one mark</u> for each soldier. The marks below represent "how many" enemy soldiers were in the patrol.

An ancient Egyptian scout would have represented the "number of" soldiers by:



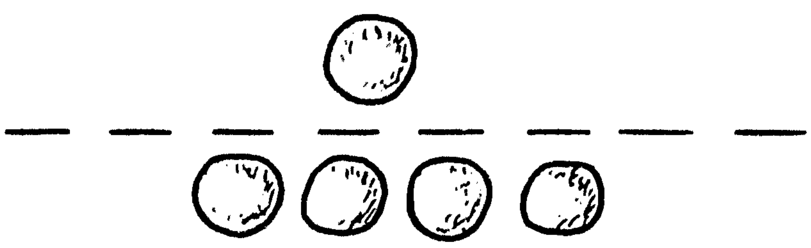
If this scout was an ancient Babylonian, the "number of" soldiers would have been represented by:



A Roman would have shown:

XXXIV

A Mayan scout would have shown:



____

____

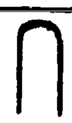
How would you show the number of soldiers in the enemy patrol? If you use our present day system, you would write:

34

Symbols were used, in each case, to represent "numbers." These symbols are called numerals.

Egyptian Numerals

Egyptian numerals date back to around 3200 B. C. Their basic set of numerals is given below. Each symbol used pictured an object. These Egyptian symbols are referred to as hieroglyphic, or picture, numerals.

OUR NUMERAL	EGYPTIAN NUMERAL	OBJECT PICTURED
1		Vertical staff
10		Heel bone
100		Coiled rope
1,000		Lotus flower
10,000		Pointing finger
100,000		Burbot fish
1,000,000		Astonished man

The Egyptians used a combination of these symbols to represent numbers. They would repeat a symbol as many times as necessary, then the number represented would be the sum of the repeated numerals. To illustrate this idea, suppose we represent 53,222 using the Egyptian numerals.

or: 50,000 + 3,000 + 200 + 20 + 2 = 53,222

Two ideas were used by the Egyptians to make these numbers easy to read:

1. Like symbols were grouped together in small groups.

2. In reading a number from left to right, symbols representing larger numbers were placed first, then symbols representing smaller numbers were placed "to the right of those" representing larger numbers.

The idea of writing from "larger to smaller" was merely for reading purposes. The numerals could have actually been placed in any order, and the value would still have been the same. For example:

| | | ∩ ∩ = 23

| ∩ | | ∩ = 23

| | ∩ | ∩ = 23

∩ ∩ | | | = 23

If the numerals are placed in a different order in our present system, is the value the same?

We could write: 23. If we rearranged the digits, we have: 32. Are these two numbers equal? No, of course they are not since:

23 "is not equal" to 32.

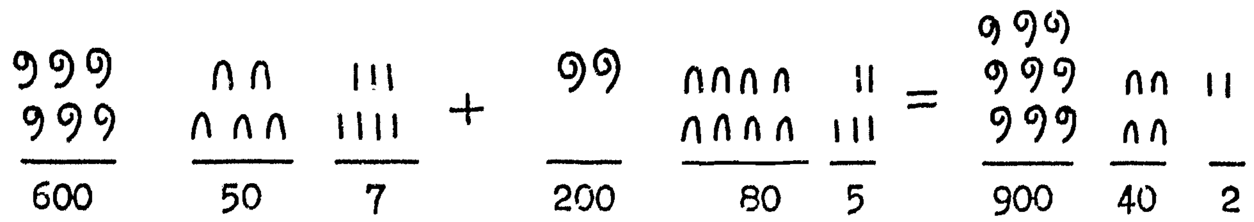
Then our system has a characteristic that the Egyptian system did not have: It is the idea of place value. The Egyptians did not have a symbol for zero either.

To build a numeration system—a system of naming numbers—the Egyptians used these ideas:

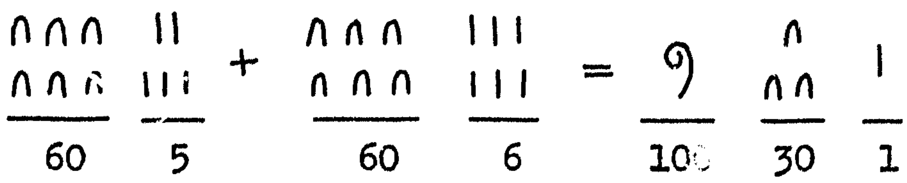
1. A basic set of numerals (symbols).
2. A single numeral (symbol) representing a collection of objects. (The Egyptian numerals show they grouped in "bundles" of tens.)
3. A repeated symbol to form other "numbers."
4. Adding a group of symbols to find the "number" represented.

Does our present system use all of these ideas? Compare the addition and subtraction examples using our numerals and Egyptian numerals.

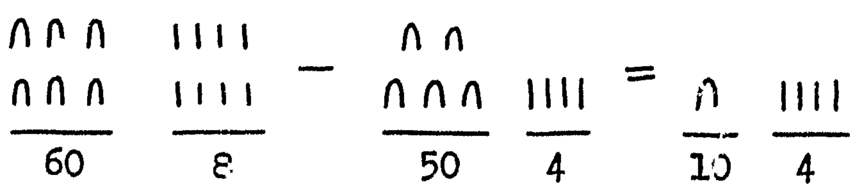
Addition:



An easier problem:



Subtraction:



A multiplication or division problem was much harder. Multiplication was performed by a "doubling and halving method." It is sometimes called the "Russian peasant method." Division was performed by reversing the multiplication procedure. See if you can find this procedure in any of your library material.

Would you agree that each of these processes is more difficult and cumbersome to perform than in our system? Part of this difficulty is because the Egyptian numeration system did not have place value.

Babylonian Numerals

Numerals used by the Babylonians date back as far as 3500 B. C. The Babylonians used soft clay to form "tablets." A stick or stylus was used by scribes to write numerals on these tablets. To preserve the writing, these tablets were baked in the sun until they were almost as hard as concrete. Tablets with part of the Babylonian multiplication tables have been discovered. One tablet begins with 18 X 1, 18 X 2, 18 X 3, and goes all the way up to 18 X 59. The complete Babylonian multiplication would have to go up to 59 X 59 as they had a base 60, or sexigesimal system. From the Babylonian system we get our divisions of 60 minutes in an hour and 60 seconds in a minute. We also get our degrees for measuring angles from their sexigesimal (sixty) system.

The table below shows the wedge-shaped numerals used by the Babylonians. This type of writing is called Cuneiform writing.

Babylonian	▽	▽▽▽▽▽	◁	◁ ▽▽	▽	◁	▽
Hindu Arabic (Our system)	1	5	10	12	60	600	3600

The examples above show two basic symbols, ▽ and ◁. By using the idea of place value these symbols could be repeated to write large numbers. The Babylonians had no zero symbol, and this made it necessary to leave "blank" spaces. Each position in the Babylonian system was "sixty" times greater than the position immediately to the right.

36			◁◁◁ ▽▽▽▽▽▽
360		▽▽▽▽▽▽	
374		▽▽▽▽▽▽	◁ ▽▽▽▽
3,673	▽	▽	◁ ▽▽▽
4,261	▽	◁ ▽	▽
36,000	◁		
36,882	◁	◁ ▽▽▽▽	◁◁◁◁ ▽▽

Activities

Fill in the missing Hindu Arabic or Babylonian numerals.

59			
	▽	▽	◁◁ ▽
		▽▽▽	◁
66			
	▽▽▽▽▽		
18,881			
		◁◁◁	
	◁◁	▽	◁◁ ▽▽▽▽▽
		◁◁◁	▽▽▽
36,820			
3,639			
	◁◁◁		◁ ▽
18,064			
		◁ ▽▽▽	
		◁◁	◁◁ ▽▽▽
	▽	◁◁	◁◁◁◁ ▽
		▽▽	◁
66			
		▽▽▽	
720			
		◁◁ ▽▽▽▽▽▽▽▽▽	◁ ▽▽▽▽▽▽▽
36,004			
		◁◁ ▽▽▽▽	◁◁◁◁◁ ▽▽
641			
		▽▽▽▽▽▽▽▽▽	◁◁◁◁◁ ▽▽▽▽▽▽▽▽▽
	▽▽▽▽▽	◁◁◁ ▽▽	◁◁◁◁ ▽

Roman Numerals

Once in awhile we still use Roman numerals. Can you think of any of the ways they are used today? Perhaps you can if you keep a "look-out" for Roman numerals.

The table below shows some of the basic symbols used by the Romans and their corresponding value in our system.

Roman system	I	V	X	L	C	D	M
Our system	1	5	10	50	100	500	1,000

Originally the Romans repeated symbols to indicate large numbers. Remember that the Egyptians had also done this. In later usage, however, some new ideas were employed. These new ideas were:

1. A Subtractive Idea--A symbol representing a smaller number is to be subtracted if written immediately to the left of a symbol representing a larger number.

Example a: Rather than write 4 as IIII, we can write 4 as IV, which is: V - I = IIII.

Example b: Rather than write 9 as VIIII, we can write 9 as IX, which is: X - I = VIIII.

The Romans used the following restrictions:

a. I before only a V or X.

b. X before only an L or C.

c. C before only a D or M.

V, L, and D never appear before a symbol representing a larger number.

Below are some examples of Roman numerals.

Roman System				Our System
XXXIV	=	10 + 10 + 10 + (5 - 1)	=	34
CMX	=	(1000 - 100) + 10	=	910
CCXC	=	100 + 100 + (100 - 10)	=	290
MMMCM	=	1000 + 1000 + 1000 + (1000 - 100)	=	3900

2. A Multiplicative Idea--A bar above a symbol to indicate 1000 times its normal value.

Roman System	"Means"	Our System	Roman System
$\overline{V}$	5 X 1000	5,000	MMMMM
$\overline{X}$	10 X 1000	10,000	MMMMMMMMMM

As you can see, this made it much easier to write large numbers.

A summary of the basic ideas of this system are:

1. A basic set of symbols.
2. These symbols could be repeated to represent numbers.
3. To find the "number" a group of symbols represented, you simply added, or subtracted, the value represented by each symbol.
4. If a symbol representing a smaller value was to the left of a symbol representing a larger value, this first value was subtracted from the larger value.
5. Large numbers were formed by placing a bar over a symbol which meant that the symbol was multiplied by 1000.

Do you notice any differences of basic ideas among the Egyptian, Babylonian, and Roman systems?

Hindu-Arabic Numerals

The Hindu-Arabic numeration system was invented by the Hindus. This system dates back to around 500 years before Christ. Arab scholars learned the system from the Hindus and were responsible for spreading its usage to other countries. This numeration system became very popular. It became a challenge to the Roman system which, at one time, was used in most of Europe.

During the 16th century the Hindu-Arabic system generally replaced the Roman system. The Hindu-Arabic system is now used throughout most of the world. Because of its "ten" grouping idea, it is called a decimal system, as "deci" means ten. Certain features of the decimal system made it superior to other numeration systems. These features are:

1. Compared to other systems, it is easy to perform calculations.
2. It is easy to express very large and very small numbers.
3. It is a place-value system using ten basic symbols.

In every historical numeration system we find the use of symbols, called numerals, used to convey the idea of number. Each system also had some type of grouping scheme. However, many ancient systems did not employ the idea of place value. This made calculations very difficult. Many systems had no symbol for zero, and, strange as it may seem, zero was the last single digit to be included in the Hindu-Arabic system. Zero was invented long after the other single digits.

To gain a better understanding of our numeration system, suppose we examine some of its basic features. Ten basic symbols are used in our system. These symbols, along with their names, are given below.

Name:	Zero	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
Symbol or Numeral:	0	1	2	3	4	5	6	7	8	9

The value assigned to each numeral depends on its position. Starting on the right with the ones' position, the positional values are given below. Each position is multiplied by ten to reach the next larger position. Names of some of the positions are also given below. Note that the dots show that this idea extends on.

Name	...	Ten thousands'	Thousands'	Hundreds'	Tens'	Ones'
Numeral	...	10,000	1,000	100	10	1
	...	(10 X 1000)	(10 X 100)	(10 X 10)	(10 X 1)	(1)

This system groups in "bundles of ten" and is often referred to as base ten. To use these numerals to express the "idea" of how many, we can group as a collection of objects in tens.

How many x's are listed below?

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x x

Grouped in tens:

x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	
x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x x x x x x x	x x x x

Now we see one group of ten <u>tens</u>, two groups of ten, and four ones. Recording under our positions we have:

(Ten tens) <u>Hundreds'</u>	<u>Tens'</u>	<u>Ones'</u>
1	2	4

In regular notation we write: 124.
An expanded form of writing numerals is used to show place value.
For 124 we can write:

$$124 = 100 + 20 + 4$$
$$124 = (1 \times 100) + (2 \times 10) + 4$$
$$124 = (1 \times 10^2) + (2 \times 10^1) + (4 \times 10^0)$$

The last form shows the use of exponents in expanded notation. Only after a student understands exponents will the last form be used. By using exponents, names of the positions can be given by using 10 with some exponent. Some of these are given below.

one	1	10^0
ten	10	10^1
hundred	10 X 10	10^2
thousand	10 X 10 X 10	10^3
ten-thousand	10 X 10 X 10 X 10	10^4
hundred-thousand	10 X 10 X 10 X 10 X 10	10^5
million	10 X 10 X 10 X 10 X 10 X 10	10^6
ten-million	10 X 10 X 10 X 10 X 10 X 10 X 10	10^7
hundred-million	10 X 10 X 10 X 10 X 10 X 10 X 10 X 10	10^8
billion	10 X 10 X 10 X 10 X 10 X 10 X 10 X 10 X 10	10^9

Some of the basic features of our numeration system are:

1. Symbols--ten single digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
2. Place value
3. Ten grouping scheme--for the whole numbers we have:

less than ten	=	ones
ten ones	=	1 ten
ten tens	=	1 hundred
ten hundreds	=	1 thousand
ten thousands	=	1 ten-thousand
ten ten-thousands	=	1 hundred-thousand
ten hundred-thousands	=	1 million
.		.
.		.
.		.

Earlier in this unit we learned that bases or "grouping schemes" other than ten have been used. The Mayans used base twenty, and the Babylonians had a base sixty. Suppose we select a base five or a base twelve rather than base ten. How would this affect our number of single digit numerals? Perhaps we should explore a situation where we group in "bundles" other than ten.

BASES TO BUSINESS

Imagine that this coming summer a student applies for a job. The student is given a job in the Shipping and Receiving Department. The new employee is given the following instructions and information. Let's see if you could handle this job.

Employee Instructions and Information

1. All items the company produces are sent to this department to be packaged and shipped.
2. You must keep a careful record of the number of items you receive and a record of each shipment. A code is used to record shipments.
3. Cost is to be figured for each shipment. Always ship at the lowest possible cost.
4. Packages are made up to hold a certain number of items. You have packages as follows:

Package Sizes

(number of items a particular package will hold)

...	One hundred twenty-five	Twenty-five	Five	One

Notice that each package size is five times larger than the next smaller package size.

COST

Package Size	Cost Per Package
One	$ 1.00
Five	4.00
Twenty-five	15.00
One hundred twenty-five	50.00
Six hundred twenty-five	100.00

Notice that is is always cheaper to ship the next larger size package than five of the smaller package sizes.

Below is a sample data sheet which is for discussion purposes. It is filled in correctly.

SAMPLE DATA SHEET

For each shipment:

Start with the largest size and record the number of each package size shipped. Continue recording to the ones' position. Use zeros to indicate no package of that size in shipment.

Number of objects received	Shipping code	Package Size: Six hundred twenty-five	One hundred twenty-five	Twenty-five	Five	Ones	Cost
18	(33) five				3	3	$15
20	(40) five				4	0	$16
10	(20) five				2	0	$ 8
5	(10) five				1	0	$ 4
87	(322) five			3	2	2	$55
149	(1044) five		1	0	4	4	$70

Before filling in a data sheet, practice "packaging" by placing a circle around items as they are to be packaged. Let X's represent items. Use package sizes to record and get code numbers. Write a small <u>five</u> after the code number to show the packaging scheme.

Received	Code	Packages
12	$(22)_{five}$	xxxxx xxxxx x x
24	$(44)_{five}$	xxxxx xxxxx xxxxx xxxxx x x x x
35	$(120)_{five}$	xxxxxxxxxxxxxxxx xxxxxxxxxxx xxxxx xxxxx
17		
20		

Activities

Using the idea just explained, complete the following data sheet.

Number of Objects	Shipping Code	Package Sizes: Six Hundred Twenty-five	One Hundred Twenty-five	Twenty-five	Five	One	Cost
19	$(34)_{five}$				3	4	$16
21							
28							
10							
89							
160							
350							
	$(23)_{five}$						
	$(124)_{five}$						
	$(12)_{five}$						

NUMBER BASES

The "packaging idea" shows how to group in bundles other than "ten." A <u>base five</u> system was illustrated in using shipping code numbers. Notice the set of single digits needed for our code numbers.

0, 1, 2, 3, 4

Notice how a number is written in base five: $(23)_{five}$. The <u>five</u> is written to show the base.

Place Value and Position

As with the base ten system, all other base systems have definite place values assigned to each numeral depending upon its place in the numeral.

Every base system begins on the right with the "ones'" position and proceeds to the left, one position at a time, with the place value a power of the indicated base.

Activities

Count around the room using base five.

Operations in Base Five

Addition:

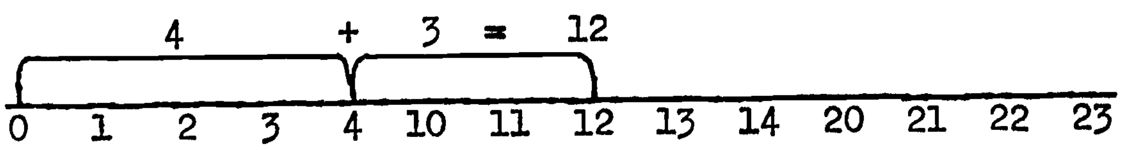
Here we need to become familiar with some of the following:

1 + 1 = 2 ("two")	2 + 4 = 11 ("one one")
2 + 2 = 4 ("four")	3 + 4 = 12 ("one two")
4 + 1 = 10 ("one zero")	4 + 4 = 13 ("one three")

Examples:

$$\begin{array}{r}13\\ +\ 4\\ \hline 22\end{array}\quad\begin{array}{r}13\\ +\ 2\\ \hline 20\end{array}\quad\begin{array}{r}13\\ +\ 3\\ \hline 21\end{array}\quad\begin{array}{r}23\\ +\ 24\\ \hline 102\end{array}\quad\begin{array}{r}234\\ +\ 243\\ \hline 1032\end{array}\quad\begin{array}{r}4223\\ +\ 342\\ \hline 10120\end{array}\quad\begin{array}{r}3213\\ +\ 322\\ \hline 4040\end{array}$$

A base five number line can be used for addition; it is shown below for adding 4 + 3. Using a strip of used adding machine paper, each student can make an individual number line.



Subtraction:

Some of the following problems will be helpful:

10 - 3 = 2 ("two") 11 - 3 = 3 ("three")

12 - 4 = 3 ("three") 14 - 3 = 11 ("one one")

10 - 1 = 4 ("four")

Examples:

$$\begin{array}{rrrrrr} 324 & 312 & 2423 & 2214 & 1203 & 1403 \\ -\ \underline{221} & -\ \underline{104} & -\ \underline{1333} & -\ \underline{1213} & -\ \underline{122} & -\ \underline{134} \\ 103 & 203 & 1040 & 1001 & 1031 & 1214 \end{array}$$

Here also a base five number line can be used for subtraction by counting backwards.

12 - 4 = 3

0 1 2 3 4 10 11 12 13 14 20 21 22 23

Multiplication:

2 X 4 = 13 ("one three") 4 X 4 = 31 ("three one")

3 X 3 = 14 ("one four") 2 X 3 = 11 ("one one")

4 X 3 = 22 ("two two")

Compare these base ten and base five multiplication problems:

$$\begin{array}{r} 12_{ten} \\ \times\ \underline{12_{ten}} \\ 24 \\ \underline{24} \\ 144_{ten} \end{array} = \begin{array}{r} 22_{five} \\ \times\ \underline{22_{five}} \\ 44 \\ \underline{44} \\ 1034_{five} \end{array} \qquad \begin{array}{r} 10_{ten} \\ \times\ \underline{10_{ten}} \\ 100_{ten} \end{array} = \begin{array}{r} 20_{five} \\ \times\ \underline{20_{five}} \\ 400_{five} \end{array}$$

$$\begin{array}{r} 25_{ten} \\ \times\ \underline{25_{ten}} \\ 125 \\ \underline{50} \\ 625_{ten} \end{array} = \begin{array}{r} 100_{five} \\ \times\ \underline{100_{five}} \\ 10000_{five} \end{array} \qquad \begin{array}{r} 9_{ten} \\ \times\ \underline{9_{ten}} \\ 81_{ten} \end{array} = \begin{array}{r} 14_{five} \\ \times\ \underline{14_{five}} \\ 121 \\ \underline{14} \\ 311_{five} \end{array}$$

BASE FIVE ADDITION TABLE

+	0	1	2	3	4	10	11	12	13	14	20	21	22
0						10							
1									14				
2		3											
3					12				22				
4									22				
10						20							
11													32
12													
13		14											
14													
20													
21													
22							31						

BASE FIVE MULTIPLICATION TABLE

X	0	1	2	3	4	10	11	12	13	14	20	21	22
0					0								
1								12					
2		2											
3													
4						40							
10									130				
11				33									
12													
13							143						
14													
20												420	
21			42										
22								314					

Activities

Perform the indicated operations using base five.

Set A

	a)	b)	c)	d)
1.	$14 + 4$	$23 + 21$	$22 + 14$	$244 + 204$
2.	$11 - 4$	$32 - 14$	$320 - 42$	$403 - 134$
3.	32×2	34×3	41×20	243×2
4.	$2\overline{)202}$	$3\overline{)333}$	$4\overline{)404}$	$21\overline{)441}$

Set B

	a)	b)	c)	d)
1.	$21 + 3$	$43 + 14$	$42 + 22$	$32 + 14$
2.	$40 - 3$	$44 - 23$	$244 - 21$	$4103 - 424$
3.	21×14	41×24	103×42	2114×23
4.	$4\overline{)422}$	$3\overline{)341}$	$4\overline{)310}$	$32\overline{)3433}$

Operations in Base Two

When men started to make electrical computors, they were faced with the problem of counting. Electricity is either <u>on</u> or <u>off</u>. So they had two numerals to work with, "off" as zero and "on" as 1. In order to make the computor work, this counting system was used:

Base ten	1	2	3	4	5	6
Base two	1	10	11	100	101	110
Computor	on	on, off	on, on	on, off, off	on, off, on	on, on, off

Using a strip of adding machine paper, make a number line for base two. These numerals are called binary numerals. "Bi" means two, and "nary" is derived from the word number. You will probably notice that binary numerals are very long and tiresome to write; but computation with numbers expressed in the binary system is very simple. Construct a base two addition and multiplication table.

Addition:

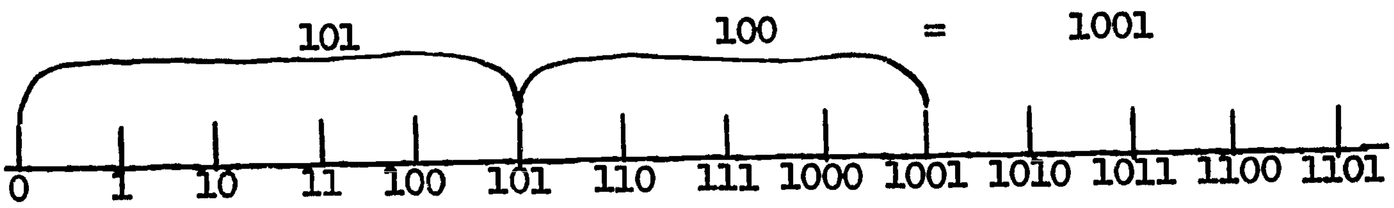
You only need to know the following additions:

1 + 0 = 1   0 + 0 = 0   0 + 1 = 1   1 + 1 = 10

Examples:

$$\begin{array}{r} 10 \\ +\ 1 \\ \hline 11 \end{array} \qquad \begin{array}{r} 111 \\ +\ 10 \\ \hline 1001 \end{array} \qquad \begin{array}{r} 1111 \\ +\ 1011 \\ \hline 11010 \end{array}$$

We may use a base two number line to determine addition problems. Add 101 + 100.



Subtraction:

To subtract you need to know the following subtraction combinations.

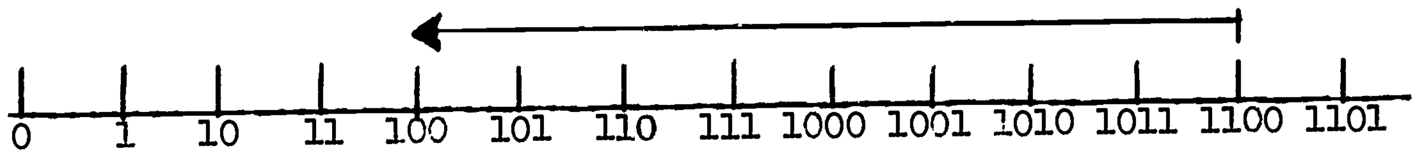
$0 - 0 = 0$ $\quad 1 - 0 = 1$ $\quad 1 - 1 = 0$ $\quad 10 - 1 = 1$

Examples:

```
   11          101        1101
 - 10         - 11       - 110
 ----         ----       -----
    1           10         111
```

Use the base two number line to help you solve the subtraction of 1000 from 1100.

1100 - 1000 = 100



Multiplication:

Become familiar with the following:

$0 \times 0 = 0$ $\quad 1 \times 0 = 0$ $\quad 0 \times 1 = 0$ $\quad 1 \times 1 = 1$

Examples:

```
   10          11         1101
 X 10        X 11        X 101
 ----        ----        -----
   00          11         1101
  10          11         0000
 ----        ----       1101
  100        1001       -------
                        1000001
```

Division:

Only two number facts are necessary for division:

```
   0        1
 1)0      1)1
```

Examples:

```
    101          110          1010
  1)101    101)11110    110)111111
    1          101           110
     0         101            111
     0         101            110
      1        ---             11
      1
```

Activities

Perform the indicated operations using binary numerals.

Set A

1. a) $\begin{array}{r} 101 \\ +\ 11 \\ \hline \end{array}$ b) $\begin{array}{r} 1011 \\ +\ 1111 \\ \hline \end{array}$ c) $\begin{array}{r} 1111 \\ +\ 1111 \\ \hline \end{array}$ d) $\begin{array}{r} 110111 \\ +\ 11101 \\ \hline \end{array}$

2. a) $\begin{array}{r} 111 \\ -\ 10 \\ \hline \end{array}$ b) $\begin{array}{r} 111 \\ -\ 101 \\ \hline \end{array}$ c) $\begin{array}{r} 11101 \\ -\ 1011 \\ \hline \end{array}$ d) $\begin{array}{r} 11000 \\ -\ 1010 \\ \hline \end{array}$

3. a) $\begin{array}{r} 11 \\ \times\ 10 \\ \hline \end{array}$ b) $\begin{array}{r} 101 \\ \times\ 11 \\ \hline \end{array}$ c) $\begin{array}{r} 111 \\ \times\ 101 \\ \hline \end{array}$ d) $\begin{array}{r} 11011 \\ \times\ 1101 \\ \hline \end{array}$

4. a) $11\overline{)110}$ b) $10\overline{)1010}$ c) $101\overline{)11110}$ d) $110\overline{)111011}$

Set B

1. a) $\begin{array}{r} 10 \\ +\ 11 \\ \hline \end{array}$ b) $\begin{array}{r} 101 \\ +\ 10 \\ \hline \end{array}$ c) $\begin{array}{r} 1011 \\ +\ 101 \\ \hline \end{array}$ d) $\begin{array}{r} 11011 \\ +\ 1101 \\ \hline \end{array}$

2. a) $\begin{array}{r} 101 \\ -\ 11 \\ \hline \end{array}$ b) $\begin{array}{r} 111 \\ -\ 101 \\ \hline \end{array}$ c) $\begin{array}{r} 1101 \\ -\ 1011 \\ \hline \end{array}$ d) $\begin{array}{r} 10011 \\ -\ 1100 \\ \hline \end{array}$

3. a) $\begin{array}{r} 11 \\ \times\ 11 \\ \hline \end{array}$ b) $\begin{array}{r} 111 \\ \times\ 10 \\ \hline \end{array}$ c) $\begin{array}{r} 1011 \\ \times\ 101 \\ \hline \end{array}$ d) $\begin{array}{r} 10010 \\ \times\ 1111 \\ \hline \end{array}$

4. a) $10\overline{)110}$ b) $11\overline{)1011}$ c) $111\overline{)11011}$ d) $101\overline{)10111}$

Counting in Other Bases

Ten Based System	Two Based System	Five Based System
1	1	1
2	10	2
3	11	3
4	100	4
5	101	10
6	110	11
7	111	12
8	1000	13
9	1001	14
10	1010	20
11	1011	21
12	1100	22
13	1101	23
14	1110	24
15	1111	30
16	10000	31
17	10001	32
18	10010	33
19	10011	34
20	10100	40
21	10101	41
22	10110	42
23	10111	43
24	11000	44
25	11001	100

Since most of us have ten fingers and are not controlled by electrical impulses, we do not need to use base five or base two in our computations. The only real benefit from all this is that we understand our base ten numeration system better. Which do you prefer—base ten, base five, or base two?

NUMERATION

ILLUSTRATION OF TERMS

Base - A way of grouping numbers; base 10 groups numbers by tens, for example.

Bi - A part of a word which means two.

Binary - A synonym for base two. Any number can be expressed by a combination of 0's and 1's using groups of size two.

Cuneiform writing - Wedge-shaped writing used by the Babylonians and some ancient cultures.

Decimal system - A synonym for base ten; a system of expressing any number by combinations of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 which group by tens.

Degrees - The unit by which angles are measured.
Example: An angle which measures 30 degrees.

Expanded form - 532 written as $(5 \times (10 \times 10)) + (3 \times 10) + (2 \times 1)$ is said to be written in expanded form.

Exponents - A shorthand device used by mathematicians to indicate the number of times a number is to be used as a factor.

Halving and doubling method - A procedure of finding a product which is done by halving one factor and doubling the other; it is often called the Russian peasant method.

Hieroglyphics - the picture numerals used by the Egyptians and some other ancient cultures in order to represent numbers; each symbol actually pictured an object.

Notation - A system of abbreviations, signs, or figures used to save time and space.

Numbers - A number is an idea--of how many, which one, etc. Symbols (numerals) are used to convey this idea.

Numerals - A word or symbol used to represent a number.
Examples: 0, 1, 2, IV, X.

Numeration System - A systematic way to name numbers. ..

Examples: Egyptian Hieroglyphics

Hindu-Arabic system

Place value - The idea that a digit will represent a certain number because of the place it occupies.

Example: 23 In 23 the 2 represents 2 groups of 10.

203 In 203 the 2 represents 2 groups of 100.

Quinary system - A system of numeration consisting of grouping by five; another name for base 5.

Russian peasant method - A halving and doubling method of multiplying.

Sexagesimal system - A system of numeration consisting of grouping by sixty.